

NAG Toolbox for MATLAB

f08ql

1 Purpose

f08ql estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix.

2 Syntax

```
[s, sep, m, info] = f08ql(job, howmny, select, t, vl, vr, mm, 'n', n)
```

3 Description

f08ql estimates condition numbers for specified eigenvalues and/or right eigenvectors of a real upper quasi-triangular matrix T in canonical Schur form. These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = ZTZ^T$ (with orthogonal Z), from which T may have been derived.

f08ql computes the reciprocal of the condition number of an eigenvalue λ_i as

$$s_i = \frac{|v^H u|}{\|u\|_E \|v\|_E},$$

where u and v are the right and left eigenvectors of T , respectively, corresponding to λ_i . This reciprocal condition number always lies between zero (i.e., ill-conditioned) and one (i.e., well-conditioned).

An approximate error estimate for a computed eigenvalue λ_i is then given by

$$\frac{\epsilon \|T\|}{s_i},$$

where ϵ is the *machine precision*.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to λ_i , the function first calls f08qf to reorder the eigenvalues so that λ_i is in the leading position:

$$T = Q \begin{pmatrix} \lambda_i & c^T \\ 0 & T_{22} \end{pmatrix} Q^T.$$

The reciprocal condition number of the eigenvector is then estimated as sep_i , the smallest singular value of the matrix $(T_{22} - \lambda_i I)$. This number ranges from zero (i.e., ill-conditioned) to very large (i.e., well-conditioned).

An approximate error estimate for a computed right eigenvector u corresponding to λ_i is then given by

$$\frac{\epsilon \|T\|}{sep_i}.$$

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – string

Indicates whether condition numbers are required for eigenvalues and/or eigenvectors.

job = 'E'

Condition numbers for eigenvalues only are computed.

job = 'V'

Condition numbers for eigenvectors only are computed.

job = 'B'

Condition numbers for both eigenvalues and eigenvectors are computed.

Constraint: **job** = 'E', 'V' or 'B'.

2: **howmny** – string

Indicates how many condition numbers are to be computed.

howmny = 'A'

Condition numbers for all eigenpairs are computed.

howmny = 'S'

Condition numbers for selected eigenpairs (as specified by **select**) are computed.

Constraint: **howmny** = 'A' or 'S'.

3: **select**(*) – logical array

Note: the dimension of the array **select** must be at least $\max(1, n)$ if **howmny** = 'S', and at least 1 otherwise.

Specifies the eigenpairs for which condition numbers are to be computed if **howmny** = 'S'. To select condition numbers for the eigenpair corresponding to the real eigenvalue λ_j , **select**(*j*) must be set **true**. To select condition numbers corresponding to a complex conjugate pair of eigenvalues λ_j and λ_{j+1} , **select**(*j*) and/or **select**(*j* + 1) must be set to **true**.

If **howmny** = 'A', **select** is not referenced.

4: **t**(ldt,*) – double array

The first dimension of the array **t** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The *n* by *n* upper quasi-triangular matrix *T* in canonical Schur form, as returned by f08pe.

5: **vl**(ldvl,*) – double array

The first dimension, **ldvl**, of the array **vl** must satisfy

if **job** = 'E' or 'B', **ldvl** $\geq \max(1, n)$;
if **job** = 'V', **ldvl** ≥ 1 .

The second dimension of the array must be at least $\max(1, mm)$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'

If **job** = 'E' or 'B', **vl** must contain the left eigenvectors of *T* (or of any matrix QTQ^T with *Q* orthogonal) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vl**, as returned by f08qk or f08pk.

If **job** = 'V', **vl** is not referenced.

6: **vr**(ldvr,*) – double array

The first dimension, **ldvr**, of the array **vr** must satisfy

if **job** = 'E' or 'B', **ldvr** $\geq \max(1, n)$;
if **job** = 'V', **ldvr** ≥ 1 .

The second dimension of the array must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'

If **job** = 'E' or 'B', **vr** must contain the right eigenvectors of T (or of any matrix QTQ^T with Q orthogonal) corresponding to the eigenpairs specified by **howmny** and **select**. The eigenvectors **must** be stored in consecutive columns of **vr**, as returned by f08qk or f08pk.

If **job** = 'V', **vr** is not referenced.

7: **mm – int32 scalar**

The number of elements in the arrays **s** and **sep**, and the number of columns in the arrays **vl** and **vr** (if used). The precise number required, m , is n if **howmny** = 'A'; if **howmny** = 'S', m is obtained by counting 1 for each selected real eigenvalue, and 2 for each selected complex conjugate pair of eigenvalues (see **select**), in which case $0 \leq m \leq n$.

Constraint: $\mathbf{mm} \geq \mathbf{m}$.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **t**.

n , the order of the matrix T .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldt, ldvl, ldvr, work, ldwork, iwork

5.4 Output Parameters

1: **s(*) – double array**

Note: the dimension of the array **s** must be at least $\max(1, \mathbf{mm})$ if **job** = 'E' or 'B' and at least 1 if **job** = 'V'.

The reciprocal condition numbers of the selected eigenvalues if **job** = 'E' or 'B', stored in consecutive elements of the array. Thus **s**(j), **sep**(j) and the j th columns of **vl** and **vr** all correspond to the same eigenpair (but not in general the j th eigenpair unless all eigenpairs have been selected). For a complex conjugate pair of eigenvalues, two consecutive elements of **s** are set to the same value.

s is not referenced if **job** = 'V'.

2: **sep(*) – double array**

Note: the dimension of the array **sep** must be at least $\max(1, \mathbf{mm})$ if **job** = 'V' or 'B' and at least 1 if **job** = 'E'.

The estimated reciprocal condition numbers of the selected right eigenvectors if **job** = 'V' or 'B', stored in consecutive elements of the array. For a complex eigenvector, two consecutive elements of **sep** are set to the same value. If the eigenvalues cannot be reordered to compute **sep**(j), then **sep**(j) is set to zero; this can only occur when the true value would be very small anyway.

If **job** = 'E', **sep** is not referenced.

3: **m – int32 scalar**

m , the number of elements of **s** and/or **sep** actually used to store the estimated condition numbers. If **howmny** = 'A', **m** is set to n .

4: **info** – **int32** scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **howmny**, 3: **select**, 4: **n**, 5: **t**, 6: **ldt**, 7: **vl**, 8: **ldvl**, 9: **vr**, 10: **ldvr**, 11: **s**, 12: **sep**, 13: **mm**, 14: **m**, 15: **work**, 16: **ldwork**, 17: **iwork**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed values sep_i may over estimate the true value, but seldom by a factor of more than 3.

8 Further Comments

For a description of canonical Schur form, see the document for f08pe.

The complex analogue of this function is f08qy.

9 Example

```
job = 'Both';
howmny = 'All';
select = [false];
t = [0.7995, -0.1144, 0.006, 0.0336;
     0, -0.0994, 0.2478, 0.3474;
     0, -0.6483, -0.0994, 0.2026;
     0, 0, 0, -0.1007];
vl = [1, 0, 0, 0;
      -0.1101757724599591, 0.8492971136777001, 0, 0;
      -0.02369735945664465, 0, 0.5250761453388553, 0;
      -0.01052671448401023, -0.2630232022148211, 0.7369767977851788, 1];
vr = [1, 0.06811593844054897, 0.02369735945664464, 0.008112867113990377;
     0, 0.6182478862610575, 0, 0.2206494686069181;
     0, 0, 1, -1;
     0, 0, 0, 0.7124731021612292];
mm = int32(4);
[s, sep, m, info] = f08ql(job, howmny, select, t, vl, vr, mm)
```

```
s =
    0.9937
    0.7028
    0.7028
    0.5711
sep =
    0.6252
    0.3743
    0.3743
    0.3125
m =
         4
info =
         0
```

